

Spreadsheet Collection User's Guide for  
Elementary Mathematical Models 2<sup>nd</sup> Edition  
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## 1. Overview

This collection of spreadsheets is intended to provide an easy way for students to define and study various families of growth models (also known as difference equation models) quickly and easily. In particular, after entering parameter values, the user will be immediately provided with a table of sequence terms and a graph. There is also an option to enter data values to which a model can be compared. Almost no knowledge of the Excel program is necessary. The spreadsheets should be compatible with versions of Excel from 2010 or later.

There are three types of spreadsheets in the collection. First there is a single spreadsheet for defining and graphing a sequence. Second, there are six spreadsheets, each designed for use with one of the families of growth models considered in the text. Finally, there are two spreadsheets that facilitate a trial-and-error approach to fitting a curve to a set of data.

Every excel file in the collection has two or more separate pages or worksheets. When first accessed from the website, each spreadsheet opens on the first of its pages. Users should remember that there are other pages available, and that they can be accessed by selecting tabs at the bottom of the program window.

On each spreadsheet, numerical or text values appear in a grid of boxes, referred to as *cells*. A standard coloring convention has been adopted in all the spreadsheets in the collection: users are intended to enter data only into cells that are colored yellow. Also, in the upper left hand corner of each spreadsheet there is a box labelled “Directions: hover mouse here.” Placing the hotspot of the mouse in that box reveals the directions.

Comments specific to each file in the collection are provided below, organized according to the three groups described above.

## 2. Defining and Graphing a Sequence (`seqgraph.xlsx`)

The first page of this file allows the user to enter several terms of a sequence and automatically displays the graph. On the second page the user may enter up to three different sequences and compare their terms and graphs.

This spreadsheet is intended to complement Chapter 1. For students who are familiar with excel or who are confident of their computer skills, the spreadsheet’s cell duplication methodology may provide some insight about recursive and direct equations for sequences. Other students may prefer to interact with this spreadsheet simply by entering numerical values. None of the other files in the collection depend on

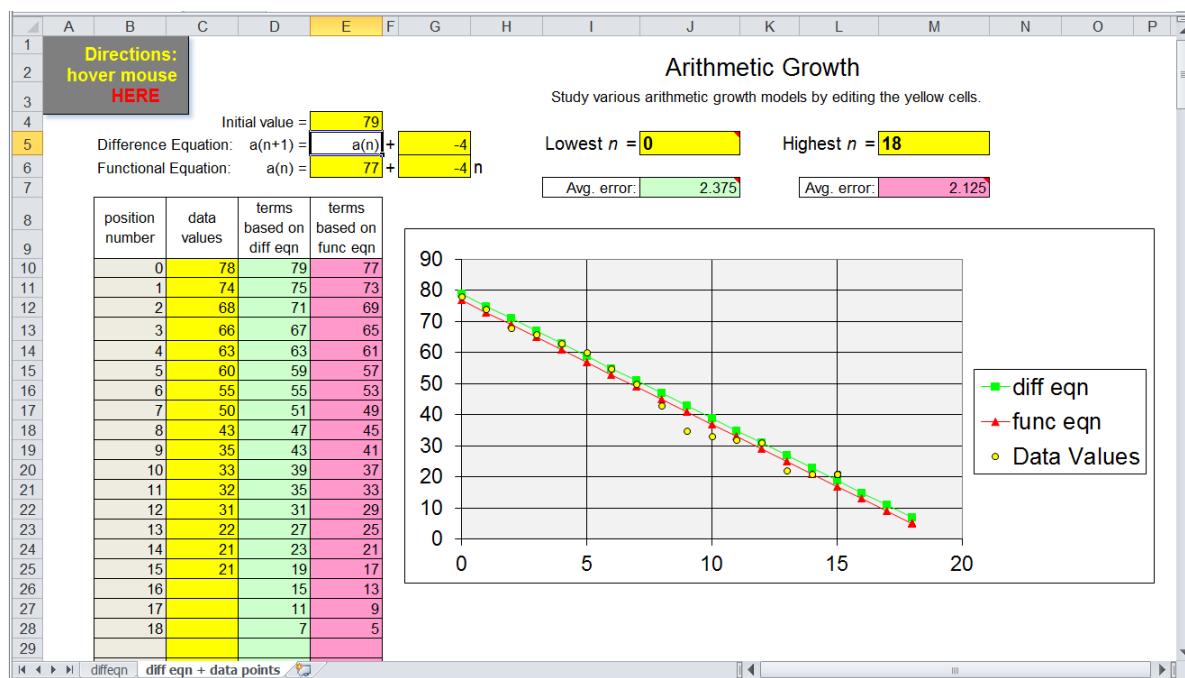
using cell duplication. Users who are not already familiar with cell duplication and who wish to delve into that topic will find additional information at the end of this document.

### 3. General Information About Spreadsheets for Specific Families of Growth Models

Separate files are provided for arithmetic growth, quadratic growth, geometric growth, mixed growth, logistic growth, and refined logistic growth. In each file, when initially accessed from the webpage, the first page is displayed. This allows the user to enter the parameters (generally an initial term and certain coefficients from the difference equation), and obtain a table of sequence terms and a graph. Except in the case of logistic growth, the user must specify the parameters for the difference equation and functional equation separately. This enables visual verification that both equations produce the same graph, and reinforces how the coefficients of each equation are related to each other. Users can also vary parameters to observe the effect on the graph.

The second page of the file allows the user to also enter sequence terms corresponding to observed data. The graph will display both the observed data and the terms produced by the difference and functional equations for the model. The spreadsheet also displays the average absolute error between observed data terms and computed sequence terms. This serves as an indication of how closely the model approximates the observed data.

Following a trial-and-error approach, the user can adjust model parameters to reduce the average absolute error as far as possible. Moreover, by adjusting the parameters only for the difference equation, it is possible to see on the screen graphs for two versions of the model. If the newer version is found to be better than the prior version, the functional equation can be updated to agree with the new version. If not, the prior version of the difference equation can be restored. For this purpose, the *undo* function in Excel is handy. An example with arithmetic growth is shown in the figure below.



In the figure, the initial value for the difference equation is 79, whereas the constant term in the functional equation is 77. On the average, the functional equation graph (in red) seems to be a better fit for the data points (in yellow) than the difference equation graph (in green). This is consistent with the average error values. The average error for the difference equation, shown in the green cell, is higher than the average error for the functional equation, shown in the pink cell.

#### 4. Additional Information About Specific Excel Files

Three of the difference equation model excel files have some additional pages to assist in formulating models to fit observed data. These are detailed below.

**Mixed Growth (mixedgrowth.xlsx).** For **mixed growth sequences**, successive ratios of first differences are all the same. This will not necessarily occur for observed data, but nearly constant ratios may be an indication that a mixed model will be appropriate. Moreover, this nearly constant value can be used to define the  $r$  parameter of the approximating mixed model, and the first differences can give clues about possible values of the  $d$  parameter. For these reasons, the mixed growth sequence excel file (mixedgrowth.xlsx) includes a third page displaying first differences and their ratios for data points a user enters on the second page.

**Logistic Growth (logisticgrowth.xlsx).** To formulate a **logistic growth** sequence approximating a set of data, it is necessary to define a linear growth factor model. This is supposed to estimate the growth factor for a particular data value as a linear function of the data value. Given data values  $d_0, d_1, d_2, \dots$ , the corresponding growth factors are  $r_0 = \frac{d_1}{d_0}, r_1 = \frac{d_2}{d_1}, r_2 = \frac{d_3}{d_2}, \dots$ . Plotting the points  $(d_0, r_0), (d_1, r_1), (d_2, r_2), \dots$ , we attempt to find a straight line that is as close as possible to all the points. The logistic growth excel file (logisticgrowth.xlsx) provides a page to assist in this process. The third page of the file (*find m and L*) computes and tabulates the growth factors for a given set of data points, and shows the points  $(d_0, r_0), (d_1, r_1), (d_2, r_2), \dots$ , on a graph. The user enters parameters defining a linear equation, and the corresponding line is also shown on the graph. These can be modified through a trial-and-error process to obtain as close a fit as possible to the plotted points. The page also automatically calculates the  $m$  and  $L$  parameters for a logistic growth difference equation based on the approximating line. These in turn can be used to define the logistic growth sequence on page 2 of the excel file.

**Refined Logistic Growth (refined\_logistic.xlsx).** Refined logistic growth is very similar to logistic growth, except that the linear model is defined to approximate the reciprocal growth factors, rather than the growth factors. In the refined logistic growth excel file (refined\_logistic.xlsx), there is an extra page for finding the linear equation fitting points of the form  $(d_0, 1/r_0), (d_1, 1/r_1), (d_2, 1/r_2), \dots$ , very similar to the extra page in the logistic growth excel file.

There is a fourth page in the refined logistic growth excel file for finding a logistic function to approximate a data set. This can be applied without formulating a difference equation model, as shown in an example in the text. This is useful if the  $d$  values in a data set are for irregularly spaced times, making consistent growth factors difficult to compute. Regularly spaced data points are not necessary when we fit a logistic function directly to the data.

## 5. Fitting a Curve to a Data Set

Two of the excel files are designed for fitting functions of various kinds to a set of data points. Here, the object is to use a visual trial-and-error approach to find curves that are close to the data points. An average error is displayed for each approximating curve. This can be used to fine-tune the trial-and-error process. These excel files are described below.

**Linear Equations (lineqn.xlsx).** This excel file has three pages, for the three standard forms of a linear equation: slope-intercept, point-slope, and two-intercept. On each page users enter data points in the yellow columns, and parameters for defining a linear equation. The data points are shown on a graph, along with a straight line defined by the linear equation. In addition, computed y coordinates are shown for each data point in the table, as well as the errors. The error for each data point is found by subtracting its y coordinate from the y value computed using the linear equation. The average absolute error appears below the graph.

**Nonlinear Equations (nonlineqn.xlsx).** This excel file is very similar to the one just described, but it is used to fit nonlinear functions to data sets. There are six pages, for the following kinds of functions: quadratic, base  $b$  exponential, base  $e$  exponential, shifted base  $b$  exponential, shifted base  $e$  exponential, and logistic. For each function type there is a standard equation with parameters (also known as coefficients) to be defined by the user. As with the linear equations excel file, the user enters data points and parameters, and then observes a graph showing both the data points and the defined function. The spreadsheet shows error values for each data point, and the average absolute error appears below the graph. The page for logistic functions is the same as the final page of the refined logistic growth excel file (refined\_logistic.xlsx).

## 6. Detailed Instructions for Duplicating Cells in Excel

**Cell Definition.** Excel allows the entry of a cell to be computed using other cells. For example, consider the small tables illustrated below. We can define the cell A3 to be the sum of the entries in cells A1 and A2 as follows:

- Click cell A3
- Type the = key
- Click cell A1
- Type the + key
- Click cell A2
- Type the enter key.

	A	B
1	3.2	
2	6.4	
3		

Before

	A	B
1	3.2	
2	6.4	
3		9.6

After

This attaches the formula  $A1+A2$  to cell A3, and displays 9.6 in the cell. Changing the values in cells A1 and A2 result in a corresponding change to cell A3.

**Relative Addressing.** We can copy cell A3 and paste it into another location. In this context, the cells in the formula  $A1+A2$  are interpreted in relation to the location of A3. That is, the formula is interpreted

to mean “the sum of the cells located one and two positions above A3”. Thus, if we copy A3 and paste it into A4, the result displayed will be the sum of the two cells directly above A4, as shown in the figure at right.

	A	B
1	3.2	
2	6.4	
3	9.6	
4	16	

We can also copy a cell into a range of cells. Copying A3 into cells A4 through A8 would produce a table in which, starting with A3, each entry is the sum of the two entries above it. Moreover, there is a shortcut for copying and pasting in this way. Observe that cell A4 in the top figure at right is highlighted with a black border and that there is a little square in the lower right-hand corner. The border and the little square appear when cell A4 is clicked. The square is a handle that can be used to duplicate the cell into adjacent cells. To do so, click and hold the little square, drag it vertically or horizontally (or both), and then release it.

This duplicates A4 to every square in the range defined by the dragging operation. The bottom figure at right illustrates the result of dragging the handle from A4 down to A8.

	A	B
1	3.2	
2	6.4	
3	9.6	
4	16	
5	25.6	
6	41.6	
7	67.2	
8	108.8	

For this example, the duplication process provides a quick way to generate a sequence in which each term (after the first two) is the sum of the two preceding terms. By similar methods, sequences can be defined according to any pattern that relates one term to one or more preceding terms. In seqgraph.xlsx, there are columns for both the sequence terms as well as the position numbers. This permits implementing patterns in which terms of the sequence depend on prior terms, or on position numbers, or both. Patterns that depend only on position numbers are *direct*, and correspond to *functional equations*. Patterns that depend on one or more preceding terms are *recursive* and correspond to *difference equations*.